Homework 10: Complexity Theory

Due: November 25, 2025

Problem 1. Compute the 3-CNF equivalent of the Boolean expression $x_1 \cap (x_3 \cup x_2 \cup \bar{x_1} \cup x_6) \cap (x_2 \cup \bar{x_3}) \cap (x_7 \cup x_3 \cup \bar{x_1})$. Prove that the two expressions are Boolean equivalent.

Problem 2.

A vertex cover of a graph $G = (V, E)$ is a set of vertices $D \subseteq V$ that includes at least one endpoint of every edge of the graph.

The k-vertex-cover problem is the decision problem: does a graph $G(V, E)$ have a vertex cover of size k.

Prove the k-vertex-cover problem is NP-complete. [Hints: prove that G has a k-vertex-cover iff G has an independent set of size $|V| - k$.]

Problem 3. Given a set of numbers $S = \{s_1, \ldots, s_n\}$, the PARTITION problem is to decide whether there is a set $T \subset S$, such that $\sum_{s \in T} = \sum_{s \in S \setminus T}$. Prove that the PARTITION problem is NP-complete. [Hist: Reduce SUBSET-SUM(X, t) to PARTITION. Add one new number q to S such that there is a partition of $X \cup \{q\}$ iff there is a solution to SUBSET-SUM(X, t).]