Homework 11: Complexity Theory

Due: December 4th, 2025

Problem 1. Let Maximum-Clique be the following problem: INSTANCE: a graph G (given by an adjacency list), and a number k QUESTION: does the graph G have a clique of size $\geq k$?

1. Suppose that you have a black-box that solves the Clique problem in $O(1)$-time. Give an efficient algorithm which, for any input graph G, finds the maximum clique in G.
2. Clearly state the (asymptotic) time complexity of the algorithm and the number of queries made to the black-box.

Problem 2. Let G be a complete weighted graph in a metric space.

1. A Minimum Bottleneck Spanning Tree (ST) in G, $MBST(G)$, is a spanning tree that minimize the maximum weight of edges in the tree,

$$\text{MBST}(G) = \arg \min_{T \in \mathcal{T}(G)} \max_{e \in T} w(e),$$

   Show that a minimum (in sum of weights) spanning tree in G is also a minimum bottleneck spanning tree in G.

2. A Minimum Bottleneck TSP in G, $MBTSP(G)$, is a cycle in G that visits each vertex exactly once and minimizes the maximum weight of an edge on the cycle. Design a 3-approximation algorithm for the Minimum Bottleneck TSP problem.

Problem 3. A HITTING-SET problem is defined on a set of items U, and a collection of subsets $S_1, \ldots, S_n \subseteq U$. The problem is to find the smallest subset $T \subseteq U$ such that $T \cap S_i \neq \emptyset$ for $i = 1, \ldots, n$. Design a polynomial time $O(\log n)$-approximation for the HITTING-SET problem.