

Homework 3:

Due: October 2nd, 2025 at 2:30p.m.

This homework must be typed in $\text{\LaTeX}$ and submitted via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except where indicated, consider every problem as asking for a proof.

Problem 1.

1. Consider a partition of the vertices of a graph $G = (V, E)$ into two disjoint subsets U and W , $U \cup W = V$. Let e be an edge of minimum weight across the partition. Prove that there is a minimum spanning tree of G containing edge e .
2. Formulate and analyze a greedy algorithm for minimum cost spanning tree based on the above property.

Solution.

□

Problem 2. Tianren *really* likes **reversible** words, that is words that read the same forwards and backwards (e.g, racecar, kayak, level). He likes them so much, that he has started to try and find reversible words in any strings that he sees.

1. Given a string with n characters from the English alphabet (you may assume that they are lower case), design an efficient dynamic program that finds the longest substring (a consecutive sequence of characters) of the input string containing a *reversible* word.
2. Provide proof of the correctness of the algorithm and analyze its time complexity and memory utilization.

Solution.

□

Problem 3. Design a Huffman code for this paragraph. What is the number of bits required to encode this paragraph using the Huffman code vs. the standard ASCII code (8 bits per character)?

Solution.

□