Homework 4:

Due: October 9th, 2025 at 2:30p.m.

This homework must be typed in LaTeX and submitted via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before your write. Except where indicated, consider every problem as asking for a proof.

Problem 1. Suppose the symbols a, b, c, d, e occur with frequencies 1/2, 1/4, 1/8, 1/16, 1/16, respectively.
(a) What is the Huffman encoding of the alphabet?
(b) If this encoding is applied to a file consisting of 1,000,000 characters with the given frequencies, what is the length of the encoded file in bits?

Solution. □

Problem 2.
We use Huffman's algorithm to obtain an encoding of alphabet $\{a, b, c\}$ with frequencies f_a, f_b, f_c. In each of the following cases, either give an example of frequencies (f_a, f_b, f_c) that would yield the specified code, or explain why the code cannot possibly be obtained (no matter what the frequencies are).

- (a) Code: 0, 10, 11
- (b) Code: 0, 1, 00
- (c) Code: 10, 01, 00

Solution. □

Problem 3. Given an n-bit binary integer, design a divide-and-conquer algorithm to convert it into its decimal representation. For simplicity, you may assume that n is a power of 2.

1. Provide a succinct (but clear) description of your algorithm, including pseudocode.
2. Prove the correctness of your algorithm.
3. Analyze the running time of your algorithm. Assume that it is possible to multiply two decimal integers numbers with at most m digits in $O(m^{\log_2 3})$ time.

Hint: An n-bit binary integer x can be expressed as $x = (x_{n-1}, x_{n-2}, \cdots, x_1, x_0)_2$ where $x_i \in \{0, 1\}$. Let $x_\ell = (x_{n/2-1}, x_{n/2-2}, \cdots, x_1, x_0)_2$ be the $(n/2)$-bit binary integer corresponding to the $(n/2)$ least significant digits of x. Let $x_m = (x_{n-1}, x_{n-2}, \cdots, x_{n/2+1}, x_{n/2})_2$ be the $(n/2)$-bit binary integer representing the $(n/2)$ most significant digits of x. Then, $x = x_\ell + 2^{n/2} \cdot x_m$. This should suggest us a way to set up a divide and conquer strategy... :) Careful about the number of subproblems!

Solution. $\square$