Problem 1. Let $A$ and $B$ be sequences of points in the Cartesian plane such that the elements of $A$ (resp., $B$) are the coordinates of the corners of a convex polygon $A$ (resp., $B$).

That is, if $|A| = k$, the convex polygon is such that there is an edge connecting $A[i]$ to $A[(i + 1) \mod k]$, for $0 \leq i \leq k - 1$. The same holds for $B$.

Design an algorithm that given $A$ and $B$ constructs a sequence $C$ such that the elements of $C$ are the coordinates of the corners of a convex polynomial $C$ which is a convex hull for the set of points given by the union of the corners of $A$ and $B$. Your algorithm should run in time $O(|A| + |B|)$.

Describe your algorithm, a simplified analysis of its correctness, and analyze its running time.

Solution.
**Problem 2.** In the fuzzy pattern-matching problem given a text $T$ and a pattern $P$, we are looking to verify if the pattern $P$ appears in the text $T$. However, we are accepting as valid matches occurrences of $P$ in $T$ for which, at most, one character is mismatched. E.g., let $T = aaaaaab$ and $P = abc$, there is a fuzzy match of $P$ in $T$ for $abd$ with at most one mismatch.

1. Show how to modify the Rabin-Karp algorithm for this problem. *Besides for the initialization phase*, the algorithm should run in the expected time $O(n)$.

2. Let $\Sigma$ denote the set of possible symbols. What is the expected running time of the algorithm when including the initialization?

*Solution.*
Problem 3. Hammad is mixing paint for his house. Since his favorite color is green he’s mixing some blue paint and yellow paint together with some paint thinner. He’s managed to create $n$ different shades of green with all the blue and yellow paint that he has.

For example, suppose he made three different shades of green.

<table>
<thead>
<tr>
<th>% Compound</th>
<th>Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$</td>
</tr>
<tr>
<td>Yellow</td>
<td>0.7</td>
</tr>
<tr>
<td>Blue</td>
<td>0.2</td>
</tr>
<tr>
<td>Paint Thinner</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Then it is possible to produce a shade of green that is 35% yellow and 27.5% blue by mixing the shades he currently has in a 1 : 2 : 1 ratio (25% $S_1$, 50% $S_2$, 25% $S_3$). However, it is impossible to create the shade of green which is 20% yellow and 10% blue.

Design an $O(n \log n)$ algorithm that checks whether it’s possible to create a liquid with the specified percentage of yellow and blue. Argue the correctness of your algorithm.

**Example Input:** $[(0.7, 0.2), (0.3, 0.1), (0.1, 0.7)], (0.35, 0.275)$

**Output:** True

**Hint:** What is this nonsense about paint colors? I wonder! I guess possible ratios of Yellow and Blue used to obtain the $n$ shades of green look like coordinates of points on the plane... :)

Solution.